

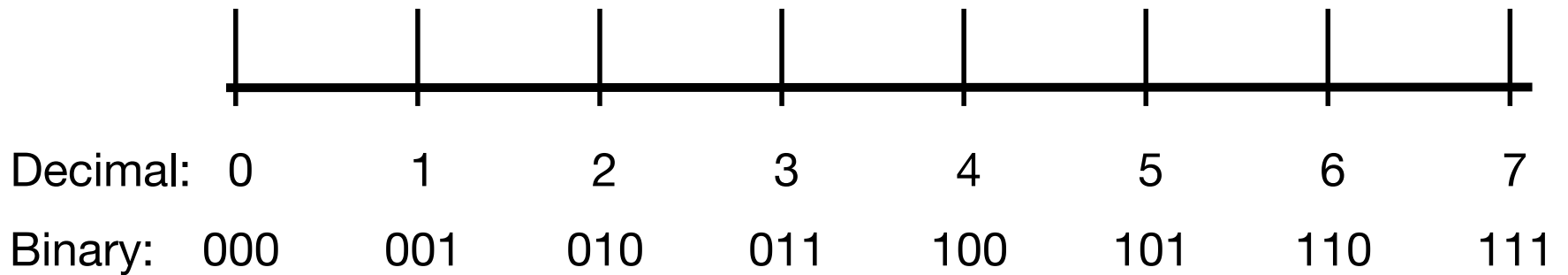
**CSE 260M / ESE 260**  
**Intro. To Digital Logic & Computer Design**

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&  
Jim Feher

# Last Time

- Binary
- *Unsigned* Integers: Extension of Place-value notation used in decimal
  - Fixed width Binary (e.g., 3-bit; 4-bit; 32-bit) forms a modular ring
  - Addition rules are simple

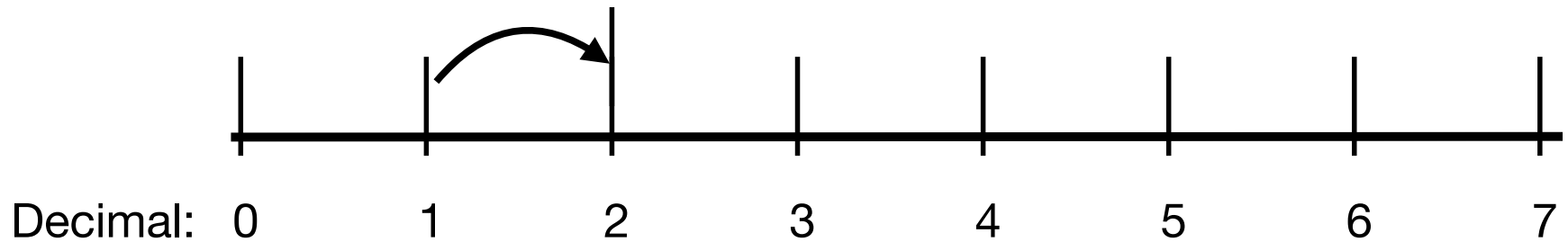
# Binary Basics: Number Line



# Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

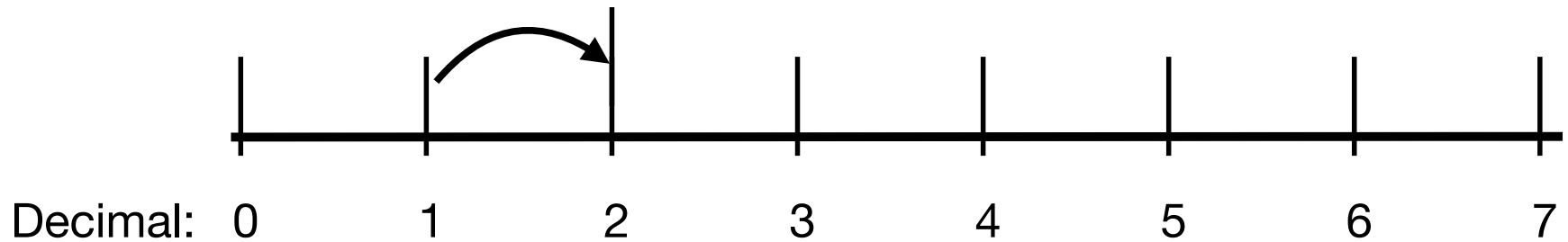
Ex:  $1+2$



# Decimal Addition: Bunch of Rules

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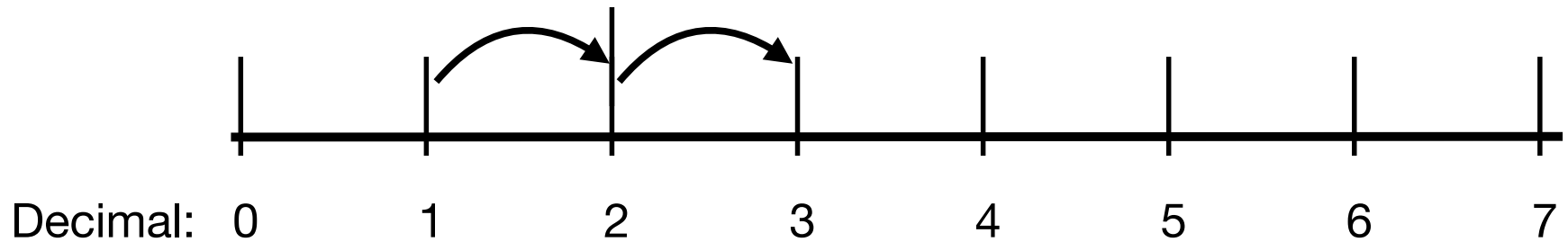
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# Decimal Addition: Bunch of Rules

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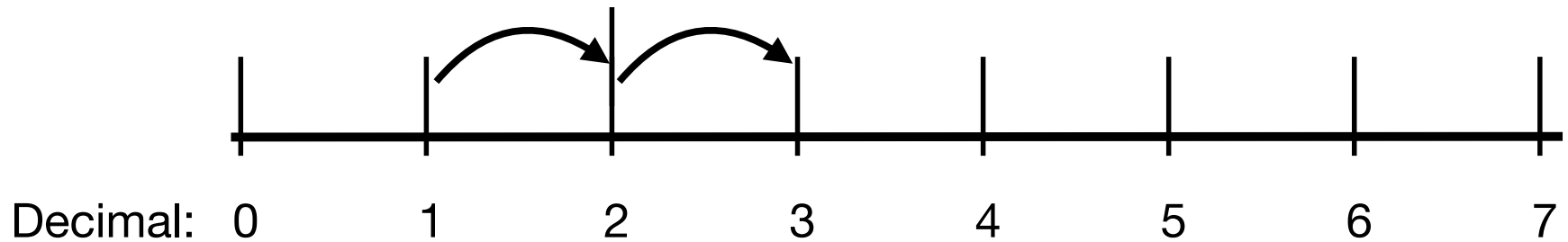
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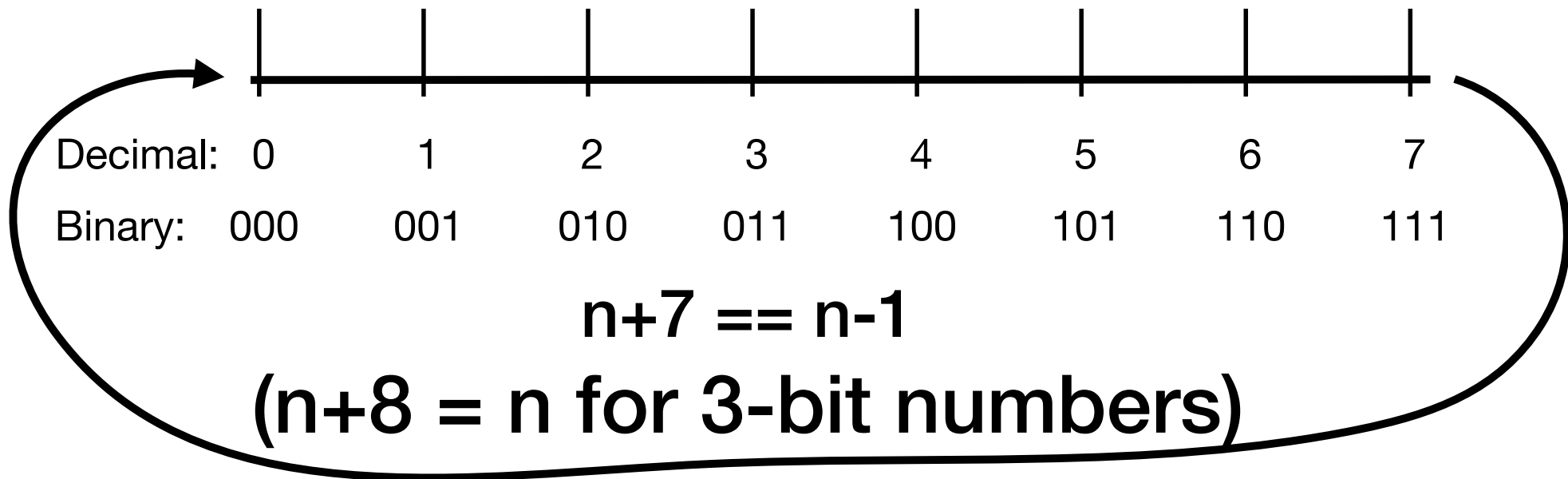
# Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

Ex:  $1+2$



# Challenge: Describe the result of $n+7$





# Chapter 2 Sections

1. Intro.
2. Boolean Equations
3. Boolean Algebra
4. From Logic to Gates

# 2.1 Intro: Combinational Logic

- (Purely) Combine inputs to produce outputs
  - Output depends *only* on current input, not pasts inputs
- Behavior of all combinational logic can be described with a table

# Binary Addition Rules: Fully Elaborated

0+ 0+ 0	=	00
0+ 0+ 1	=	01
0+ 1+ 0	=	01
0+ 1+ 1	=	10
1+ 0+ 0	=	01
1+ 0+ 1	=	10
1+ 1+ 0	=	10
1+ 1+ 1	=	11

# Binary Addition Rules: Inputs

Carry	A	B	=	Sum
0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

# Binary Addition Rules: & Outputs

Carry In	A	B	=	Carry Out	Sum
0+	0+	0	=	0	0
0+	0+	1	=	0	1
0+	1+	0	=	0	1
0+	1+	1	=	1	0
1+	0+	0	=	0	1
1+	0+	1	=	1	0
1+	1+	0	=	1	0
1+	1+	1	=	1	1

# Combinational Logic vs. Sequential Logic

- Output of Sequential Logic
  - Depends on current inputs and *sequence* of past inputs (values and order)
  - Requires concept of memory

## 2.2 Boolean Equations - History

- George: Mathematical Analysis of Logic
- Formal, algebraic approach to manipulation of binary concepts
- So?
  - Provide formal approach to manipulate concepts

# Boolean Algebra

**Table 2.1** Axioms of Boolean algebra

	Axiom		Dual	Name
A1	$B = 0 \text{ if } B \neq 1$	A1'	$B = 1 \text{ if } B \neq 0$	Binary field
A2	$\bar{0} = 1$	A2'	$\bar{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3'	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4'	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5'	$1 + 0 = 0 + 1 = 1$	AND/OR



# Boolean Algebra

**Table 2.2** Boolean theorems of one variable

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1'	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2'	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3'	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5'	$B + \overline{B} = 1$	Complements

# Boolean Algebra

**Table 2.3** Boolean theorems of several variables

	Theorem		Dual	Name
T6	$B \bullet C = C \bullet B$	T6'	$B + C = C + B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	T7'	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$	T8'	$(B + C) \bullet (B + D) = B + (C \bullet D)$	Distributivity
T9	$B \bullet (B + C) = B$	T9'	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \bar{C}) = B$	T10'	$(B + C) \bullet (B + \bar{C}) = B$	Combining
T11	$(B \bullet C) + (\bar{B} \bullet D) + (C \bullet D)$ $= (B \bullet C) + (\bar{B} \bullet D)$	T11'	$(B + C) \bullet (\bar{B} + D) \bullet (C + D)$ $= (B + C) \bullet (\bar{B} + D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \dots}$ $= (\bar{B}_0 + \bar{B}_1 + \bar{B}_2 \dots)$	T12'	$\overline{B_0 + B_1 + B_2 \dots}$ $= (\bar{B}_0 \bullet \bar{B}_1 \bullet \bar{B}_2 \dots)$	De Morgan's Theorem

## 2.4 Gates

- Not just electronics:
  - Scientific American, Vol. 258, No. 4 (APRIL 1988), pp. 118-121 (4 pages)
- Claude: Thesis

# **Simple Examples: Logic.ly**

# **Timing & Simulation**

# Demos of Circuits in JLS

- Overview of parts / ideas
  - Equation:  $D = A * B * C$ 
    - Realization A & Testing
    - Realization B B & Testing
  - Bubble Pushing
  - DeMorgan's Laws?

# Questions

- “Do we need to memorize all the boolean theorems? pls no”
- “What is the purpose of our existence?”
- [What’s the delay for and/or?]
- [What’s a minterm?] : More next week!
- [What about this DeMorgan’s stuff?]

# Next Time

- **Studio: Watch email for email / Canvas notifications about Piazza and Studio**
  - May require minor setup
  - Location may be different
- **Next Week / Reading: Rest of Chapter 2**