CSE 260M / ESE 260
Intro. To Digital Logic & Computer Design

Bill Siever
&
Jim Feher

Last Time

- Binary
- *Unsigned* Integers: Extension of Place-value notation used in decimal
  - Fixed width Binary (e.g., 3-bit; 4-bit; 32-bit) forms a modular ring
  - Addition rules are simple

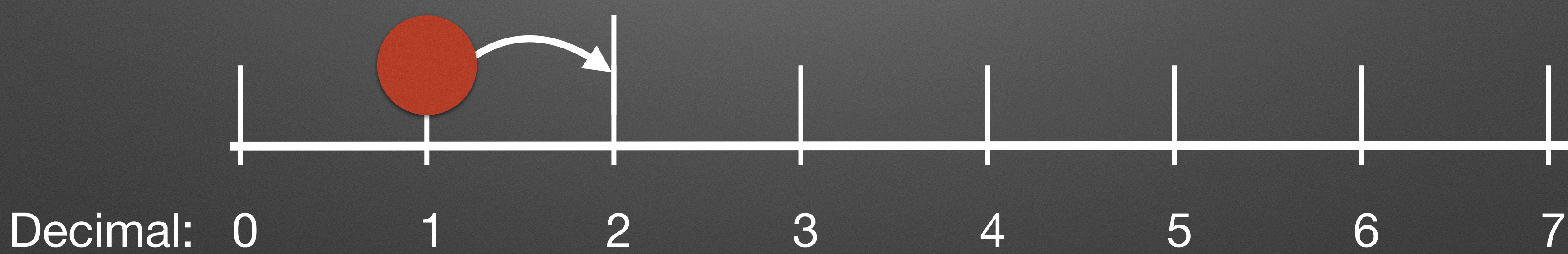
Binary Basics: Number Line

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

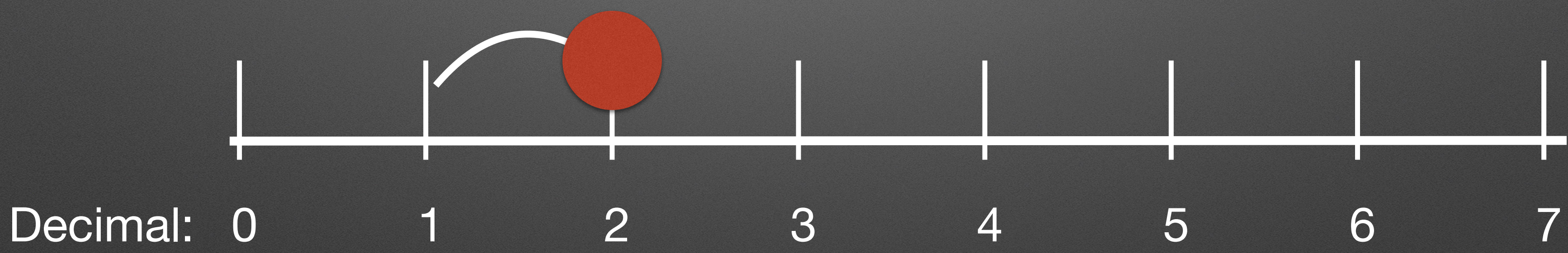
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just "encode" moving right on the number line

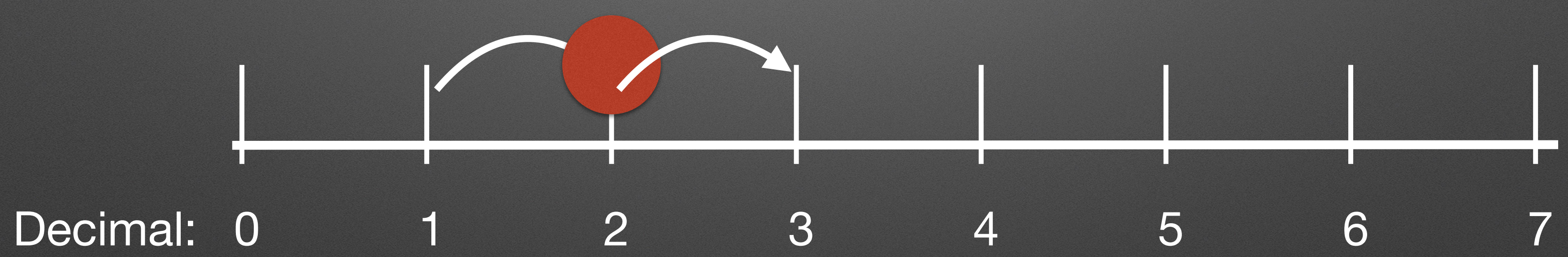
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just "encode" moving right on the number line

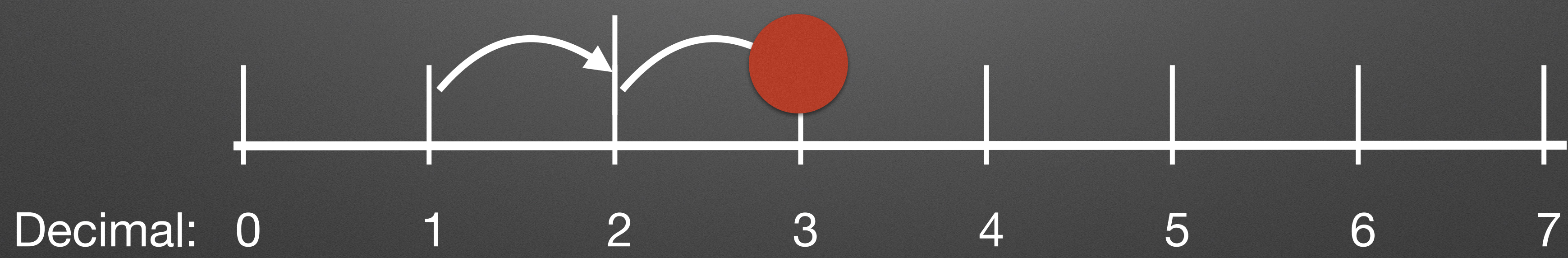
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

Ex: 1+2



Challenge: Describe the result of n+7

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

$$n+7 == n-1$$

$$(n+8 = n \text{ for 3-bit numbers})$$

Chapter 2 Sections

1. Intro.
2. Boolean Equations
3. Boolean Algebra
4. From Logic to Gates

2.1 Intro: Combinational Logic

- (Purely) Combine inputs to produce outputs
  - Output depends *only* on current input, not pasts inputs
- Behavior of all combinational logic can be described with a table

Binary Addition Rules: Fully Elaborated

0+ 0+ 0	=	00
0+ 0+ 1	=	01
0+ 1+ 0	=	01
0+ 1+ 1	=	10
1+ 0+ 0	=	01
1+ 0+ 1	=	10
1+ 1+ 0	=	10
1+ 1+ 1	=	11

Binary Addition Rules: Inputs

Carry	A	B		Sum
0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

Binary Addition Rules: & Outputs

Carry In	A	B		Carry Out	Sum
0+	0+	0	=	0	0
0+	0+	1	=	0	1
0+	1+	0	=	0	1
0+	1+	1	=	1	0
1+	0+	0	=	0	1
1+	0+	1	=	1	0
1+	1+	0	=	1	0
1+	1+	1	=	1	1

Combinational Logic vs. Sequential Logic

- Output of Sequential Logic
  - Depends on current inputs <u>and</u> *sequence* of past inputs (values and order)
  - Requires concept of memory

2.2 Boolean Equations - History

- George: Mathematical Analysis of Logic
- Formal, algebraic approach to manipulation of binary concepts
- So?
  - Provide formal approach to manipulate concepts

Boolean Algebra

Table 2.1 Axioms of Boolean algebra

	Axiom		Dual	Name
A1	$B = 0$ if $B \neq 1$	A1′	$B = 1$ if $B \neq 0$	Binary field
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$	AND/OR

Boolean Algebra

Table 2.2 **Boolean theorems of one variable**

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

Boolean Algebra

Table 2.3 Boolean theorems of several variables

	Theorem		Dual	Name
T6	$B \bullet C = C \bullet B$	T6′	$B + C = C + B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	T7′	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$	T8′	$(B + C) \bullet (B + D) = B + (C \bullet D)$	Distributivity
T9	$B \bullet (B + C) = B$	T9′	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	T10′	$(B + C) \bullet (B + \overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D)$ $= (B \bullet C) + (\overline{B} \bullet D)$	T11′	$(B + C) \bullet (\overline{B} + D) \bullet (C + D)$ $= (B + C) \bullet (\overline{B} + D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots}$ $= (\overline{B}_0 + \overline{B}_1 + \overline{B}_2 \ldots)$	T12′	$\overline{B_0 + B_1 + B_2 \ldots}$ $= (\overline{B}_0 \bullet \overline{B}_1 \bullet \overline{B}_2 \ldots)$	De Morgan's Theorem

2.4 Gates

- Not just electronics:
  - Scientific American, Vol. 258, No. 4 (APRIL 1988), pp. 118-121 (4 pages)
- Claude: Thesis

Simple Examples: Logic.ly

Timing & Simulation

Demos of Circuits in JLS

- Overview of parts / ideas
  - Equation: D = A*B*C
    - Realization A & Testing
    - Realization B B & Testing
  - Bubble Pushing
  - DeMorgan's Laws?

Questions

- “Do we need to memorize all the boolean theorems? pls no”
- “What is the purpose of our existence?”
- [What’s the delay for and/or?]
- [What’s a minterm?] : More next week!
- [What about this DeMorgan’s stuff?]

Next Time

- Studio: Watch email for email / Canvas notifications about Piazza and Studio
  - May require minor setup
  - Location may be different
- Next Week / Reading: Rest of Chapter 2