CSE 260M / ESE 260
Intro. To Digital Logic & Computer Design

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&
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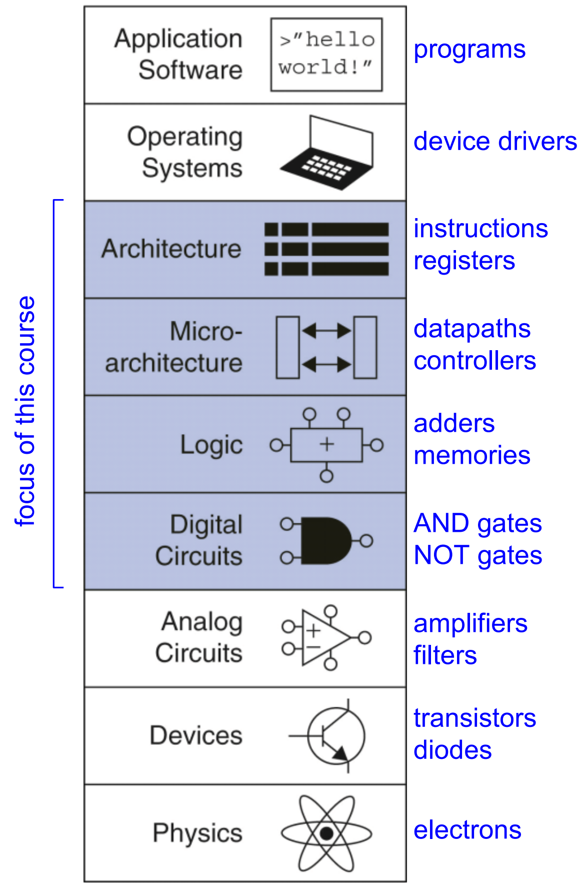
Chapter 1 Sections

1. The Game Plan
2. Managing Complexity
3. Digital Abstraction
4. Number Systems
5. Logic Gates

Course

But Architecture before Micro



Abstraction

- Digital discipline
  - Discrete values
    - Moreover, *binary* (0/1; false/true; Off/On; 0v/3v; No/Yes; …)
      - Smallest unit of information: a binary digit. Also-know-as a *Bit*
  - (Mostly) Starting at gate level

Counting

Decimal
0
1
2
3
4
5
6
7
8
9
10

Counting

Decimal
00
01
02
03
04
05
06
07
08
09
10

Counting

Decimal	Binary
00	
01	
02	
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0
01	
02	
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0
01	1
02	
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0
01	1
02	?
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	00
01	01
02	10
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	0111
08	
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	0111
08	1000
09	
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	0111
08	1000
09	1001
10	

Counting

Decimal	Binary
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	0111
08	1000
09	1001
10	1010

Binary Basics: Number Line

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Conversions

Place Value: Base 10
Example: 123

Digits	1	2	3
Place Value	100	10	1
Place Value In terms of Base	10^2	10^1	10^0
Expansion	1×10^2	$+2\times10^1$	$+3\times10^0$

Place Value: Base 2
Example: 110_2 (or 3’b110)

Digits	1	1	0
Place Value (Decimal)	4	2	1
Place Value In terms of Base	2^2	2^1	2^0
Expansion	1×2^2	$+1\times2^1$	$+0\times2^0$

Easy Conversion: Binary to Decimal

Place Value (Decimal)	128	64	32	16	8	4	2	1
Place Value In terms of Base	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Problem: What is the decimal value of 5’b10011

Place Value (Decimal)	128	64	32	16	8	4	2	1
Place Value In terms of Base	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Easy Conversion: Decimal to Binary
Greedy Algorithm Approach: Right to Left

1. Start with value n

2. Find the exponent, k, of the *largest* power of 2 that is *smaller* than n.
   (I.e., first power of 2 that can be subtracted without going negative)

3. For k down to 0:

   1. If $2^k \leq n$

      1. Write down a 1 (and move right)

      2. $n = n - 2^k$

   2. Else

      1. Write down a 0 (and move right)

Example: Convert 27 to binary (With the greedy approach)

- First power of 2 less than 27
  - 16 (2^4)
- $n = 27 - 16 = 11$
- $n = 11 - 8 = 3$
- $n = 3 - 2 = 1$
- $n = 1 - 1 = 0$

Place Value	128	64	32	16	8	4	2	1
Place Value	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Result				**1**	**1**	**0**	**1**	**1**

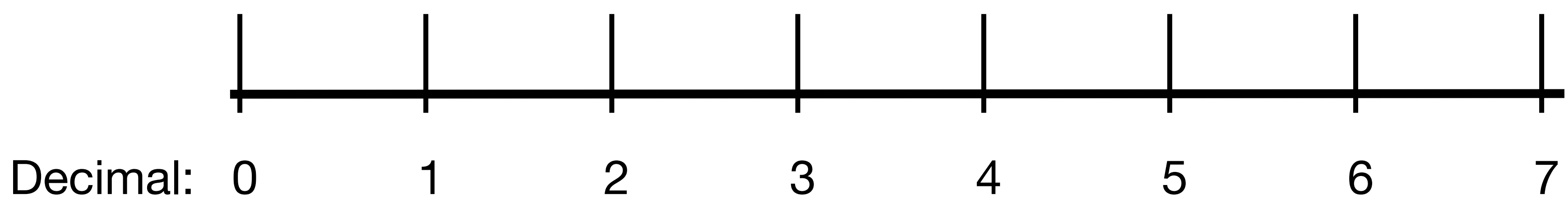
Decimal Addition

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

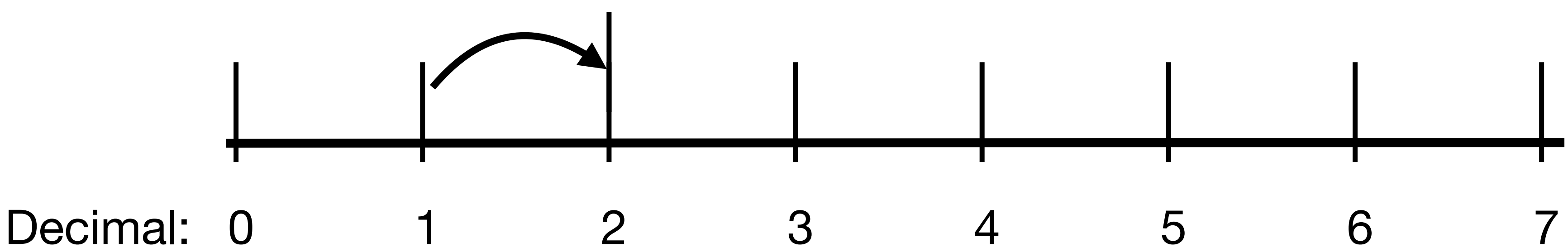
Ex: 3+4



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

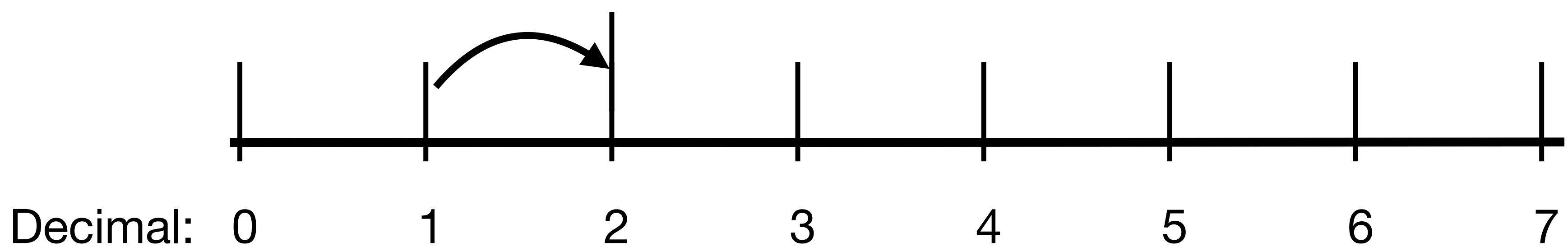
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

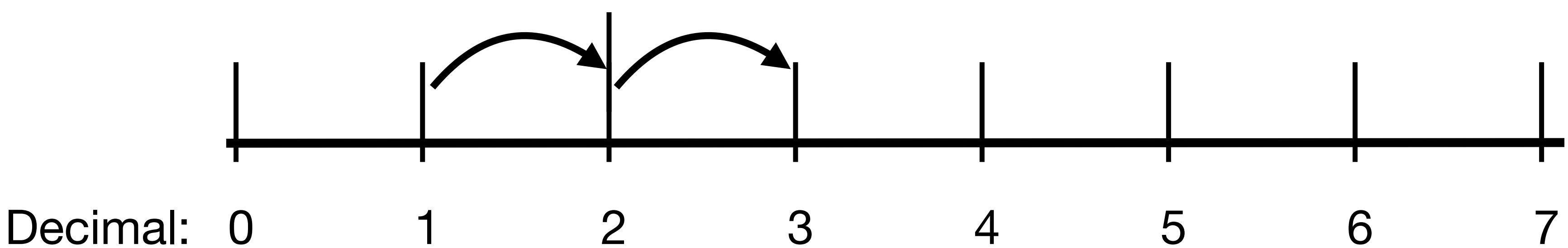
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

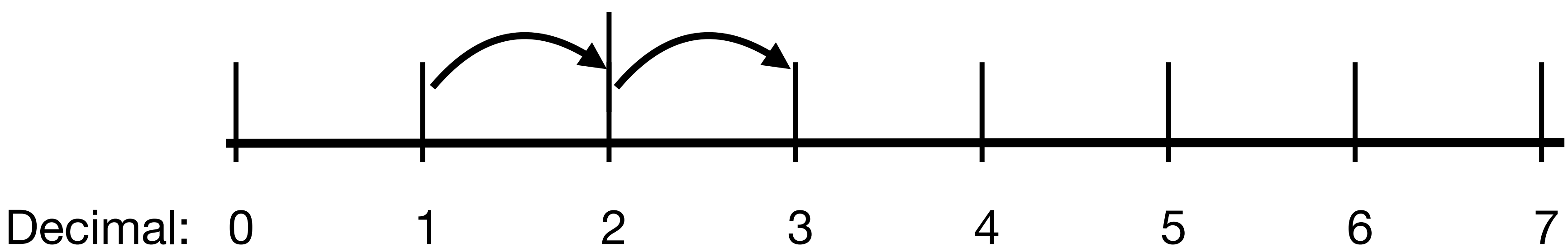
Ex: 1+2



Decimal Addition: Bunch of Rules

Rules just “encode” moving right on the number line

Ex: 1+2



Binary Addition Rules

- Condensed
  - No ones: 0+0+0 = 00
  - One one: 0+0+1 = 01
  - Two Ones: 0+1+1 = 10
  - Three Ones: 1+1+1 =11

Binary Addition Rules: Fully Elaborated

0+ 0+ 0	=	00
0+ 0+ 1	=	01
0+ 1+ 0	=	01
0+ 1+ 1	=	10
1+ 0+ 0	=	01
1+ 0+ 1	=	10
1+ 1+ 0	=	10
1+ 1+ 1	=	11

Problem

- Add 4’b1010 + 4’b0011

What’s the operation?

- Consider the following problems:
  - 123 ? 10 = 3
  - 7 ? 10 = 7
  - 29 ? 10 = 9

- Consider the following problems:
  - 123 ? 100 = 23
  - 7 ? 100 = 7
  - 29 ? 100 = 29

Why is that important?

- We’ll often work with fixed-width numbers
  - Ex: our rules of addition are just for 1 column of digits
- Multi-digit numbers are handled via chaining together fixed width operations
- Truncation to fixed width numbers is a special case of modular arithmetic (which has some cool properties)

Fixed Width / Truncation in Decimal

- We'll often work with fixed-width numbers
  - Ex: our rules of addition are just for 1 column of digits
- Multi-digit numbers are handled via chaining together fixed width operations
- Truncation to fixed width numbers is a special case of modular arithmetic (which has some cool properties)

What's the operation?

- Consider the following problems:
  - 123 ? 10 = 3
  - 7 ? 10 = 7
  - 29 ? 10 = 9

- Consider the following problems:
  - 123 ? 100 = 23
  - 7 ? 100 = 7
  - 29 ? 100 = 29

What's the operation?

- What is the 1 digit result of:
  - 122 + 1 = 3
  - 3+4 = 7
  - 15+14 = 9

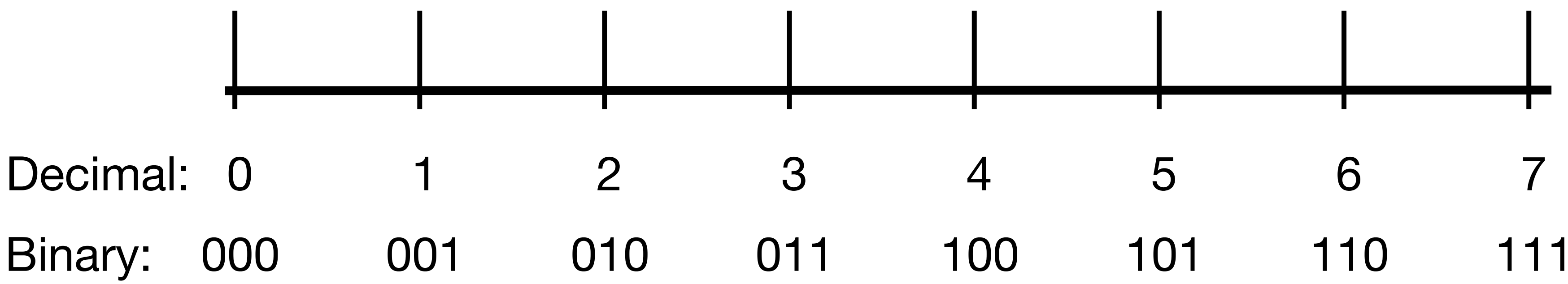
- What is the 2 digit result of:
  - 3+120 = 23
  - 2+5 = 7
  - 28+1 = 29

Modular Arithmetic & The Number Line (Binary, 3-bit)

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

What’s 1+2?

Modular Arithmetic & The Number Line (Binary, 3-bit)



What’s 6+2?

Challenge: Describe the result of n+7

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Challenge: How can you emulate n-2?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

The Magic of Modular Arithmetic: Addition can emulate subtraction!

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	—3	-2	-1

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the decimal value of the 3-bit, 2's complement numbers:

110

011

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the 3-bit, 2's complement representation of:

2

-4

-5

Hexadecimal

- Convenient, compact way to deal with binary
- Each hex digit = exactly 4 binary digits

Monday/Tuesday

- Monday by 11:59pm
  - Finish reading Chapter 1 (1.6-1.9)
  - Read Chapter 2, section 2.1-2.5
  - Do questions on Canvas by 11:59pm